Students with learning disabilities struggle with word problems in mathematics classes. Understanding the type of errors students make when working through such mathematical problems can further describe student performance and highlight student difficulties. Through the use of error codes, researchers analyzed the type of errors made by 14 sixth grade and 15 seventh grade students with language-based learning disabilities educated with either a standards-based or traditional mathematics curricula on word problems representing number and operations and algebra standards. Two main findings occurred: (a) the open-ended word problems were challenging to students with learning disabilities as demonstrated through the high rate of unanswered questions and incorrect answers, with the most frequently committed errors representing a lack of understanding of the mathematics; and (b) no relationship existed between curricula and correct or unanswered questions for sixth grade, but statistically significant relationships existed in the seventh grade, favoring the standards-based curriculum.

**Keywords:** mathematics; curriculum; error analysis

Student success in mathematics has gained considerable interest as reflected in recent federal legislation (i.e., No Child Left Behind [NCLB], 2002) and state legislation (i.e., changes in state mathematics' requirements for students) (Associated Press, 2010; Michigan Department of Education, 2006). NCLB indicated all students should be proficient in the core content areas (e.g., mathematics) and all students, including students with learning disabilities, were to be tested annually in mathematics in grades three through eight. Additionally, new state legislation is increasing what high school students are required to take in terms of mathematics courses for graduation (i.e., Algebra II) (Michigan Department of Education).

Previous research suggested students with learning disabilities struggle in the content area of mathematics, particularly as compared to their peers without disabilities (Bryant, Bryant, & Hammill, 2000; Cawley, Parmar, Foley, Salmon, & Roy, 2001; Witzel, Riccomini, & Schneider, 2008). Their struggles range from mastery of basic facts and computational fluency to problem solving (Calhoon, Emerson, Flores, & Houchins, 2007; Parmar, Cawley, & Frazita, 1996). Despite the range of mathematical challenges faced by students with learning disabilities, the primary emphasis in research has been on students' acquisition, or lack thereof, of basic facts or computational fluency, rather than problem solving (Fuchs & Fuchs, 2005).

Although problem solving is touted as the “centerpiece of mathematics reform,” mathematics assessments are typically multiple choice (Bennett & Gitomer, 2008; National Research Council, 2004; Woodward & Montague, 2002, p. 96). Problem-solving assessments are especially lacking in special education (Jitendra & Xin, 1997). Even curriculum-based measurements (CBMs), a hallmark of assessment in special education (Fuchs & Fuchs, 1993), historically measured students' procedural (i.e., rules and basic skills) rather than conceptual mathematics knowledge (Helwig, Anderson, & Tindal, 2002; Jitendra, Dupuis, & Zaslofsky, 2014).

**Problem Solving**

Problem solving is an important component of mathematics education and also a central tenet of the *Principles and Standards for School Mathematics* from the National Council of Teachers of Mathematics (Jitendra et al., 2007; NCTM, 2000; Schoenfeld, 2007). It is often placed in contradiction to more procedural knowledge, such as computation. Problem solving requires students to rely on textual information to solve a problem; in other
words, to answer a word problem students have to decipher information from the text to determine the computation (Fuchs et al., 2008). Hence, reading is a key component to solving word problems in addition to the mathematical aspects of number sense and applying the appropriate operations (Desoete, Roevers, & De Clercq, 2003).

Students with disabilities frequently struggle with solving mathematical word problems. Decades of research documented the underperformance of students with disabilities in problem solving, particularly as compared to their peers without disabilities (Algozzine, O’Shea, Crews, & Stoddard, 1987; Cawley & Miller, 1989; Cawley et al., 2001; Parmar et al., 1996). For example, Parmar and colleagues, in an analysis of word problem solving for students with and without disabilities in grades 3 through 8, found students without disabilities performed better on word problems than students with disabilities, and eighth grade students with learning disabilities struggled with even simple word problems. This struggle is particularly detrimental because word problems typically represent real-world problems and the use of mathematics in everyday life (Briars & Larkin, 1984; Jitendra, Hoff, & Beck, 1999). Having the knowledge and skills to address and solve word problems not only shows deeper and higher levels of understanding of mathematics but also the knowledge and skill to apply problem solving strategies, which are skills that transcend multiple content areas and life domains (Briars & Larkin, 1984). Several factors affect how students with disabilities solve word problems, such as perceived difficulty level of the problem, differences in solving approaches (Montague & Applegate, 1993), and language difficulties (Carnine, 1997; Parmar et al., 1996).

The research on mathematical problem solving and students with learning disabilities is limited and primarily focused on instruction (Hutchinson, 1993; Jitendra et al., 1999; Montague, 1992; Woodward & Montague, 2002; Xin et al., 2005). Far less research is focused on understanding the type of errors students with learning disabilities make in solving word problems. Yet, research focused on understanding students’ errors (i.e., error analysis) is important for several reasons: (a) error analysis can further describe student performance in problem solving extending beyond just a correct/incorrect analysis (Balacheff, 1990; Jitendra & Kameenui, 1996); (b) related, error analysis can identify strategies students use to solve word problems (Hartman, 2007); (c) error analysis can assist teachers in determining student difficulties in problem solving and suggest instructional strategies for addressing such challenges (Parmar et al., 1996), such as being used as an formative assessment (Dennis, Calhoun, Olson, & Williams, 2014); and (d) errors when solving word problems tend to be more representational than calculation based and there is a need and a desire by researchers to determine the type of errors made (Zawaiza & Gerber, 1993).

This project explored the errors students with learning disabilities made on open-ended problem-solving assessments (i.e., word problems when choice of answers are not provided) and compared the frequency of errors committed as well as the number of correctly answered and unanswered problems across the type of curriculum students received (i.e., standards-based vs. traditional). The research focused on two different types of curricula, increasing the possibilities for what could be learned. Standards-based curricula usually refer to a curriculum that embraced the National Council of Teachers of Mathematics (NCTM) standards (1989, 2000), and aligned with the principles of problem-centered curricula (Nie, Cai, & Moyer, 2009). Several textbooks were written in the 1990s to reflect the NCTM standards, and their production was funded by the National Science Foundation (NSF) (Senk & Thompson, 2003). Hence, these curricula are also referred to as NSF-funded curricula. The standards-based (i.e., NSF-funded) curricula tended to address conceptual understanding and problem solving in addition to learning rules and procedures; conceptual understanding is defined as “the comprehension of mathematical concepts, operations, and relations” (Kilpatrick, Swafford, & Findell, 2001, p. 116) or, in other words, “an integrated and functional grasp of mathematical ideas” (Kilpatrick et al., p. 118).

Traditional curricula tend to refer to a mathematics curriculum focused on learning rules and procedures; traditional curricula focus on mastery of basic skills and facts prior to moving to problem-solving (Battista, 2001; Cawley, 2002; Hudson, Miller, & Butler, 2006). Critics refer to traditional curricula as a drill and grill approach to teaching and learning mathematics (Shenk & Thompson, 2003). Note, in recent years publishers of traditional curricula discussed how the materials align with the standards, but standards-based in this article—consistent with the convention of the field—refers to those funded by NSF and focus on building conceptual understanding (Nie et al.). Stated differently, traditional curricula focus more on procedural knowledge—defined as “the ability to execute action sequences to solve problems” (Rittle-Johnson, Siegler, & Alibali, 2001, p. 346)—while standards-based curricula focus more on conceptual knowledge—defined as “an implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain” (Rittle-Johnson et al., p. 346–347).
The inclusion of the two main perspectives of middle school mathematics curricula—standards-based and traditional—broadened this study, allowing for type of curriculum received to be an additional factor in understanding word problem errors. This particular research project sought to answer the following research questions: (a) what type of errors do students with learning disabilities make on open-ended problem-solving questions?, and (b) how do the errors of students with learning disabilities differ across different curricula models (standards-based vs. traditional)?

Method

Participants

This study consisted of 14 sixth grade students and 15 seventh grade students, which represented 9.6% and 7.5%, respectively, of the total students who participated in the larger research project. All students with learning disabilities received the mathematics curriculum of the class they were placed in by their school; no assignment to a class or a curriculum was made by researchers. Students with learning disabilities in each grade represented 60.7% and 57.7%, respectively, of all students with disabilities in the larger study. All students identified with learning disabilities had a language-based learning disability (i.e., reading, written expression). Learning disabilities was defined in this particular state as “a severe discrepancy between achievement and ability that is not correctable without special education and related services” (Michigan Department of Education [DOE], 2002). All students were determined to be eligible for special education services under the category of learning disability prior to the State’s rule change allowing students with learning disabilities to be diagnosed when they fail to respond to “scientific research-based procedures” (DOE, 2009, p. 17). Per the definition, all students had average IQs and below average achievement relative to reading or written expression (see Bouck & Kulkarni, 2009, for additional information). Note, the majority of students with language-based learning disabilities had reading disabilities.

The 14 sixth-grade students with learning disabilities, ranging in age from 12 to 13, were educated in six inclusive mathematics classrooms. Eight received the standards-based mathematics curriculum, and six the traditional curriculum. The majority of participating students were female (n = 8). Thirteen students were Caucasian; one was Hispanic. Participants’ average score on the state’s general large-scale assessment in mathematics the previous year was 2.43 for those in the standards-based curriculum and 2.20 for those educated with the traditional curriculum, of which a 4 indicates an apprentice level, a 3 a basic level, 2 means a student met state standards, and 1 that a student exceeded state standards.

The 15 seventh grade students with learning disabilities, ranging in age from 13 to 14, were from five inclusive mathematics classes. Eight received the standards-based mathematics curriculum and seven the traditional. The majority of the students with learning disabilities in the seventh grade were male (n = 11). Thirteen of the students were Caucasian, one was black, and one identified as multiracial. Of the 15 students, the average score on the state's general large scale assessment in mathematics the previous year was 2.14 for those who received the standards-based curriculum and 2.33 for those educated with the traditional curriculum.

Setting

Four school districts in the state of Michigan participated in this study. The school districts were purposefully selected based on the mathematics curricula they used in their middle schools as well as their geographical location; all four districts were from the same county. The four schools had similar demographic features (i.e., socioeconomic status, student population). Two of the four school districts in the study used the same standards-based mathematics curriculum, although different editions. Specifically one used Connected Mathematics, formerly referred to as the Connected Mathematics Project (CMP) (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2004), and the other Connected Mathematics 2 (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006). Similarly with the traditional curriculum, both schools used the same series but different versions: Prentice Hall Middle Grades Math: Tools for Success and Prentice Hall Mathematics (Chapin, Illingworth, Landua, Masingila, & McCracken, 2000; Charles et al., 2008). Schools using each curriculum followed the scope and sequence of their particular curriculum as it related to the state standards. Similar content was presented in the two curricular approaches but the pedagogical approach differed. The traditional curriculum followed a more teacher-centered, procedural approach to teaching and learning mathematics while the standards-based curriculum embraced student-centered approaches and developing conceptual understanding.

These schools were interesting given the two curricular approaches—the standards-based and the traditional—were produced by the same publisher. Further, the Connected Mathematics curriculum is important to study
given its prominence in middle school mathematics education across the nation; it was recognized as one of five “exemplary” mathematics curricula by the U.S. Department of Education (1999), ranked first place in a curricular evaluation of 13 middle school curricula by the American Association for the Advancement of Science (AAAS, 2000), and substantially penetrated the middle school textbook market nationally (Education Market Research, 2008). Further, research conducted on Connected Mathematics for students without disabilities showed positive results, particularly on open-ended problems (e.g., Reys, Reys, Lapan, Holliday, & Wasman, 2003; Riordan & Noyce, 2001; Senk & Thompson, 2003); yet, research involving students with disabilities is lacking.

Across the four school districts, nine teachers were involved in the study, representing all teachers in each school who taught inclusive sixth or seventh grade mathematics classes. There were five sixth grade teachers: three taught the traditional curriculum and two the standards-based curriculum. Of the five sixth-grade teachers: three were female and two were male, all taught in a middle school setting, and all taught for at least five years (average 12.2 years of teaching). Four seventh-grade teachers were involved in the study: two teaching the standards-based curriculum and two the traditional curriculum. Three of the seventh-grade teachers were female, and all had been teaching for at least six years (average 10 years of teaching). Across both grades, the teachers who taught the standards-based mathematics curriculum all reported receiving at least one professional development experience related to the curriculum; none of the traditional curriculum teachers reported receiving any professional development relative to the specific curriculum implemented.

Procedure

This study represents one substantial component of a larger study that involved examining the achievement of middle school students with and without disabilities in standards-based vs. traditional mathematics curricula. The overarching study involved students taking bi-monthly assessments, alternating between multiple-choice (i.e., choices of answers are provided including the correct one) and open-ended (i.e., word problems in which no choices to answers are provided) problem-solving formats. The overall study and this specific component occurred across the entire school year (i.e., September through May).

This particular research component focused only on the open-ended problem-solving assessments and just the data from students with learning disabilities. This component is different from the larger study as it focused on just one aspect: the type of errors students with learning disabilities made when solving open-ended word problems. Students completed eight open-ended problem-solving assessments over a school year (i.e., roughly one per month). Teachers informed their students they had 10 minutes to complete each assessment, and they were to work independently on each problem. All students were allowed to use a calculator on all assessments.

Instruments

Each open-ended assessment consisted of three questions. However, some questions were designed with multiple subparts (i.e., part a, b, and c) (see Appendix for sample assessment questions). For sixth grade, the fewest number of responses on an open-ended problem-solving assessment was three and the greatest number was nine. For seventh grade, the fewest number of responses was three and the greatest number was seven. The assessments were created to align with the state’s mathematics standards for the number and operations and algebra standards for both grade levels. The decision was made to focus on these two content standards because they receive greater emphasis during sixth and seventh grades (Michigan Department of Education, 2005a). The assessments were created by researchers from released items off the state's previous large-scale general assessment. Using questions from previous state assessments ensured problems aligned to state standards and in taking the assessments students also practiced for the general large-scale state assessment. A mathematics education expert in the State reviewed all problems. Grade-appropriate teachers then reviewed sample assessments.

To create the sixth and seventh grade open-ended assessments, 24 questions at each grade level were created. These were created by exploring large-scale assessments and finding relevant open-ended questions in the two content standards, and specifically the substandards within Number and Operations and Algebra. For the sixth grade Number and Operations standards, researchers focused on three areas: multiplying and dividing with fractions; adding and subtracting integers and rational numbers; and solving decimal, percentage, and rational number problems. Questions were developed from four areas within the Algebra content standard: calculating rates; using variables, writing expressions and equations; combing like terms and representing linear functions using tables, equations, and graphs; and solving equations (Michigan Department of Education, 2005c). Researchers created seventh grade Number and Operations questions from two areas of standards: understanding and solving problems involving rates, ratios, and proportions; and computing with rationale numbers. Finally, three standards areas were covered for the seventh grade algebra assessments: understanding and applying direct proportional relationship and relating to
linear relationships; understanding and representing linear relationships; and combining algebraic expressions and solving equations (Michigan Department of Education, 2005b).

The 24 questions were randomly selected for placement on the eight assessments. While the number and operations and algebra strands of the state standards were the focus for both sixth and seventh grade, algebra had a slightly larger focus in seventh grade. Thus, 12 number and operation and 12 algebra problems were selected for random placement on the sixth grade assessments, whereas 10 number and operations and 14 algebra problems were selected for random placement on the seventh grade assessments. Each sixth and seventh grade problem-solving assessment consisted of one number and operations problem and one algebra problem. The third problem was randomly selected from a pool of both types for each grade.

Data Analysis

The primary means of data analysis for this study was error analysis. Error analysis involved analyzing the mathematical errors made by students when solving problems (Ashlock, 1986). Hong and Ehrensberger (2007) described error analysis as “an informal assessment” whose purpose is to “identify the mistakes students make on selected mathematics problems and determine how they arrived at the incorrect answer” (p. 43). More recently, Dennis et al. (2014) referred to error pattern analysis as a type of formative assessment. To analyze the open-ended problem-solving assessments, researchers coded each incorrectly solved open-ended problem on all eight assessments following error codes. Researchers first met to determine interrater reliability of error codes for each assessment and then independently coded the problems. Interrater reliability was calculated by dividing the total number of agreements by the total number of agreements plus disagreements and multiplying the obtained value by 100 for each assessment. The overall interrater reliability agreement for the sixth grade error analysis was 93% (range of 85.7%–95.1%) and for the seventh grade was 91.7% (range of 85.7%–94.9%). The first author entered all error codes into a SPSS database file.

Although multiple methods for classifying errors exist (Jitendra & Kameenui, 1996), this project adapted an error task analysis method specified by Polloway, Patton, and Serna (2005), which is similar to the approach used by Roberts (1968). Polloway and colleagues’ original error analysis had five categories: random responding (“errors are without any recognizable reason”), basic fact error (operation performed correctly but an error is made with basic facts or simple calculation error), wrong operation (the wrong operation is performed), defective algorithm (operation is not performed appropriately, such that steps may be out of sequence or inappropriately applied), and place value (aspects of place value are incorrect but facts, operations, and stages are performed correctly) (p. 356). To these initial five error codes, the research team added three additional ones: defective algorithm multi-step (only one step of a multi-step problem is addressed), no conceptual understanding (answer lacks conceptual understanding of problem being asked but answer is not random; can make sense of errors even if it does not address the given problem), and no algebra (fails to solve algebra problem using algebra or does not understand algebra is needed to solve the problem). These additional codes were developed by the researchers from both the errors found in student responses and as a means to evaluate the particular type of questions asked. For example, the researchers wanted to determine if the challenges were specifically related to algebra (i.e., no algebra) when algebraic questions were asked as well as to determine if the multi-step nature of the problems was problematic (i.e., the teachers in the study reported students with disabilities experienced the greatest challenge with multi-step problems in their classes as well as previous research suggested multi-step problems to be more challenging; Parmar et al., 1996). The codebook also included codes if the student solved the problem correctly and if the student did not answer the question (i.e., 99, representing missing data).

Descriptive statistics were used to analyze the data. First, data were combined across all eight assessments to represent the two strands per grade (i.e., number and operations and algebra). Once the total number of possible responses was found per strand, per grade (i.e., number of students times number of responses), we then calculated the frequency of missing data (i.e., unanswered questions). Next the number of unanswered questions was subtracted from the total possible and the frequency of correctly answered questions calculated. From the incorrect answers the frequency of each error code was calculated. Using Minitab, a Chi Square Goodness of Fit test was conducted for each strand in each grade to determine if the proportions for answers correct and missing answers were expected for each curriculum. Chi Square Tests (two-way table) were also conducted to determine the association between curricula and error codes for each strand per grade. However, given the low frequency counts of some error codes (i.e., multiple cells having less than five and multiple cells having less than one), this analysis could not be performed, and hence not reported.
Results

Sixth Grade

Number and operations standards. Across all questions addressing the sixth grade number and operations strands, the majority of students with language-based learning disabilities answered the problems incorrectly (78.7%). More students with learning disabilities educated in the standards-based curriculum correctly answered problems representing these standards (25.6% vs. 14.6%); however, there was no statistically significant relationship between answering questions correctly and curricula (p = 0.84) (see Table 1). When incorrect, the most common error across both curricula was no conceptual understanding (44.8% and 44.3%, standards-based vs. traditional, respectively), followed by defective algorithm (39.6% and 35.7%, respectively), and defective algorithm multi-step (7.3% and 8.6%, respectively). Few basic factor errors were made in either curriculum (2.1% and 1.4%, respectively). More responses to number and operations questions were left blank (i.e., missing) by students with learning disabilities who received the traditional curriculum (40.6%) than those who received the standards-based curriculum (29.9%), indicating students did not know the answer, did not have time to complete the problem, or did not want to do the problem. However, no statistically significant association occurred between curricula and not answering questions (i.e., missing data) (p = 0.11).

Algebra standards. Across the three sixth grade algebra standards, students with language-based learning disabilities answered the questions correctly 26.4% of the time (see Table 1). Students in the standards-based curriculum answered these types of problems correctly more often than those who received the traditional (30.8% vs. 20.7%); this relationship was not statistically significant (p = 0.14). When students did answer the

<table>
<thead>
<tr>
<th>Error Code</th>
<th>Numbers and Operations</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All LD</td>
<td>SB-LD</td>
</tr>
<tr>
<td>Correct</td>
<td>21.3%</td>
<td>25.6%</td>
</tr>
<tr>
<td></td>
<td>(out of 211)</td>
<td>(out of 129)</td>
</tr>
<tr>
<td>Incorrect</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Random response</td>
<td>5.4%</td>
<td>4.2%</td>
</tr>
<tr>
<td></td>
<td>(out of 166)</td>
<td>(out of 96)</td>
</tr>
<tr>
<td>Basic fact</td>
<td>1.8%</td>
<td>2.1%</td>
</tr>
<tr>
<td>Wrong operation</td>
<td>2.4%</td>
<td>2.1%</td>
</tr>
<tr>
<td>Defective algorithm</td>
<td>38.0%</td>
<td>39.6%</td>
</tr>
<tr>
<td>Place value</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Defective algorithm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>multi-step</td>
<td>7.8%</td>
<td>7.3%</td>
</tr>
<tr>
<td>No algebra</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>No conceptual understanding</td>
<td>44.6%</td>
<td>44.8%</td>
</tr>
<tr>
<td>Missing</td>
<td>32.5%</td>
<td>29.9%</td>
</tr>
<tr>
<td></td>
<td>(out of 322)</td>
<td>(out of 184)</td>
</tr>
</tbody>
</table>

Note: SB-LD refers to students with learning disabilities who received a standards-based curriculum, and T-LD refers to students with learning disabilities who received a traditional curriculum. All error code frequency data are based off the number of incorrect responses not the total number of correct response; thus excluding the number correct. Correct frequency data are based off the total number of responses (i.e., correct plus incorrect). Missing frequency data are based off the total number of possible responses.
problem incorrectly, the most frequent error code for those educated in the standards-based curriculum was no conceptual understanding (43.4%), followed by no algebra (41.0%), and then defective algorithm multi-step (7.2%). For students who received the traditional curriculum the most frequent error code was no algebra (38.4%), followed by no conceptual understanding (26.0%), defective algorithm (15.1%), and defective algorithm multi-step (15.1%). Across both curricula, students with learning disabilities made few basic fact errors (1.2% for standards-based vs. 1.4% for traditional).

**Seventh Grade**

**Number and operations standards.** Slightly less than half the students with language-based learning disabilities educated with the standards-based curriculum correctly answered problems reflecting the seventh grade number and operations standards (45.6%) as compared to 32.9% for those who received the traditional curriculum (see Table 2). This relationship between curriculum and correctly answering problems was statistically significant ($\chi^2(1) = 4.85, p = 0.03$). Students who received the traditional curriculum left number and operations questions blank more frequently (21.9% vs. 8.3%), which was also a statistically significant relationship ($\chi^2(1) = 7.03, p = 0.008$). The majority of the incorrect responses were coded

<table>
<thead>
<tr>
<th>Error Code</th>
<th>All LD</th>
<th>SB-LD</th>
<th>T-LD</th>
<th>All LD</th>
<th>SB-LD</th>
<th>T-LD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct</td>
<td>45.3%</td>
<td>45.6%</td>
<td>32.9%</td>
<td>25.3%</td>
<td>25.5%</td>
<td>25.0%</td>
</tr>
<tr>
<td>(out of 192)</td>
<td>(out of 110)</td>
<td>(out of 82)</td>
<td>(out of 229)</td>
<td>(out of 141)</td>
<td>(out of 88)</td>
<td></td>
</tr>
<tr>
<td>Incorrect</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(out of 105)</td>
<td>(out of 50)</td>
<td>(out of 55)</td>
<td>(out of 171)</td>
<td>(out of 105)</td>
<td>(out of 66)</td>
<td></td>
</tr>
<tr>
<td>Random response</td>
<td>6.7%</td>
<td>4.0%</td>
<td>9.1%</td>
<td>2.9%</td>
<td>2.9%</td>
<td>3.0%</td>
</tr>
<tr>
<td>Basic fact</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>4.1%</td>
<td>2.9%</td>
<td>6.1%</td>
</tr>
<tr>
<td>Wrong operation</td>
<td>1.0%</td>
<td>0%</td>
<td>1.8%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Defective algorithm</td>
<td>16.2%</td>
<td>22.0%</td>
<td>10.9%</td>
<td>0.6%</td>
<td>1.0%</td>
<td>0%</td>
</tr>
<tr>
<td>Place value</td>
<td>8.6%</td>
<td>10.0%</td>
<td>7.3%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Defective algorithm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>multi-step</td>
<td>16.2%</td>
<td>16.0%</td>
<td>16.4%</td>
<td>1.8%</td>
<td>1.9%</td>
<td>1.5%</td>
</tr>
<tr>
<td>No algebra</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>78.4%</td>
<td>78.1%</td>
<td>78.8%</td>
</tr>
<tr>
<td>No conceptual understanding</td>
<td>51.4%</td>
<td>48.0%</td>
<td>54.6%</td>
<td>12.3%</td>
<td>13.3%</td>
<td>10.6%</td>
</tr>
<tr>
<td>Missing</td>
<td>14.7%</td>
<td>8.3%</td>
<td>21.9%</td>
<td>38.9%</td>
<td>29.5%</td>
<td>49.7%</td>
</tr>
<tr>
<td>(out of 225)</td>
<td>(out of 120)</td>
<td>(out of 105)</td>
<td>(out of 375)</td>
<td>(out of 200)</td>
<td>(out of 175)</td>
<td></td>
</tr>
</tbody>
</table>

*Note:* SB-LD refers to students with learning disabilities who received a standards-based curriculum and T-LD refers to students with learning disabilities who received a traditional curriculum. All error code frequency data are based off the number of incorrect responses not the total number of correct response; thus excluding the number correct. Correct frequency data are based off the total number of responses (i.e., correct plus incorrect). Missing frequency data are based off the total number of possible responses.
as no conceptual understanding for both curricula (48.0% for standards-based and 54.6% for traditional). For those who received the standards-based curriculum, the next most frequently committed error was defective algorithm (22.0%), followed by defective algorithm multi-step (16.0%), and then place value (10.0%). Errors committed by students educated with the traditional curriculum included defective algorithm multi-step (16.4%), defective algorithm (10.9%), and then random response (9.1%). Students with learning disabilities receiving either curriculum made no basic fact errors.

Algebra standards. For the algebra standards, students with language-based learning disabilities who received either curriculum correctly answered the open-ended algebra problems with equal frequency (25.5% for standards-based and 25.0% for traditional); thus, this relationship was not statistically significant \( p = 0.94 \) (see Table 2). Across curricula, incorrect responses were primarily coded as no algebra (78.1% and 78.8%, respectively). Other error codes included no conceptual understanding (13.3% and 10.6%, respectively), basic fact (2.9% and 6.1%, respectively), and random response (2.9% and 3.0%, respectively). Almost half of the students with learning disabilities who received the traditional curriculum did not attempt problems representing the algebra standards (49.7%), as compared to 29.5% of those who received the standards-based curriculum. The relationship between curricula and missing data (i.e., unanswered questions) was statistically significant \( \chi^2(1) = 9.80, p < 0.01 \).

Discussion

This project explored the type of errors made by sixth and seventh grade students with learning disabilities—specifically language-based learning disabilities2—who received either a standards-based or traditional mathematics curriculum on assessments containing open-ended problems representing number and operations and algebra standards (NCTM, 2000). Two main findings occurred: (a) the open-ended problems were challenging to students with learning disabilities as demonstrated by the high rate of unanswered questions and incorrect answers, with the most frequently committed errors representing a lack of understanding of the mathematics needed to solve the problem; and (b) no relationship existed between curricula and correct or unanswered questions for sixth grade, but statistically significant relationships existed in the seventh grade, favoring the standards-based curriculum.

In general, across grade, curricula, and mathematical strands, students with language-based learning disabilities correctly answered open-ended problem-solving questions less than 50% of the time (exception, seventh grade students receiving a standards-based curriculum on the number and operations questions). The data suggest neither set of curriculum materials nor likely any of the current instruction in these classrooms is sufficient for students with learning disabilities to be successful with solving word problems that more closely represent the mathematics encountered in everyday life. Although the low percentage of correctly answered questions may not be surprising, as previous researchers indicated problem solving is challenging for students (Cawley et al., 2001; Parmar et al., 1996; Xin, 2003) and problem solving uses different cognitive abilities than straightforward computation (Fuchs et al., 2007), it should still raise concerns about the focus and depth in current mathematics curricula, specifically in meeting the needs of students with learning disabilities. The multi-step nature some of the problems in the assessments, which Parmar and colleagues noted as being even more challenging although representing the real-world mathematics students are likely to face (Fuchs & Fuchs, 2005), should not be justification for the low performance across classes. If one of the goals of schools is to prepare students for the world outside of school, then curriculum developers need to be more diligent in creating instructional materials that help students to be problem solvers.

Yet, we don't want to imply curriculum materials alone are the only reason for low performance on open-ended tasks. Teachers play a critical role in student learning, and the results suggest not enough is being done in classrooms to help students to be successful with word problems (Borko & Whitcomb, 2008). For example, it is common for students with disabilities to experience greater difficulty due to the language-based nature of word problems, and the students in this analysis had a learning disability related to language (Carnine, 1997; Parmar et al., 1996). Hence, a need exists to question what teachers are deliberately and intentionally doing to help students with the “reading” needed to be successful in mathematical situations. The role of reading—and teaching students to read mathematics (e.g., word problems)—needs greater exploration in understanding students’ struggle when solving word problems (Fuchs et al., 2006). Teachers need to actively work with students who present reading challenges on how to access word problems as well as work

2Throughout the discussion, the students in this study will be referred to as students with learning disabilities, although this is particularly targeting and referencing students with language-based learning disabilities.
with students on reading mathematics in general; in other words, focusing instruction on literacy in mathematics classes. The teachers in this study also spoke to the value of addressing the literacy aspects in mathematics problem solving, as most of the participating teachers felt that reading or language arts difficulties are problems for students with disabilities rather than mathematics. Literacy in mathematics is complicated and involves more than a using one quick-fix strategy as Parmar and colleagues and Xin (2007) suggested schools might be exacerbating the problem with addressing the language challenges as they tend to teach students with disabilities word cue schemas to solve open-ended problems, which fail when these types of problems use inconsistent language.

The analysis of incorrect responses to the open-ended problems showed the majority of errors students made represented no understanding of the mathematics needed to address the problem. In other words, the responses given suggest students with learning disabilities struggled to understand what the problem was asking and/or how to solve it (i.e., conceptual understanding). Thus, in addition to reading difficulties interfering with students making sense of the problems, they also lacked a conceptual understanding of the mathematics needed to solve certain types of problems. Mathematical literacy is not just knowing what four times eight is but rather knowing when four times eight can be used to answer a problem (NCTM, 2000; Organisation for Economic Co-operation and Development, 2003). The students’ lack of understanding is likely a contributing factor to the number of problems not even attempted. The data for both strands show students in sixth and seventh grade left over 20% of the possible responses unanswered, the one exception being seventh grade students in the number and operations strand who received a standards-based curriculum (8.3%). The fact that for the number and operations standards, seventh grade students receiving standards-based curriculum were statistically better in both frequency of problems solved correctly as well as frequency of problems attempted could suggest that over time with a more consistent focus on conceptual development—a key component of the standards-based curricula—students do better with open-ended problems. However, additional research would be needed to make such a determination.

The second main finding was the lack of statistically significant relationships between the two curricula and the number of correct and unanswered problems in the sixth-grade. Curriculum was not influential in terms of students correctly solving the open-ended problem-solving assessments, contrary to what was hypothesized. Given the nature of the standards-based curriculum involving more problem solving, it is somewhat surprising sixth-grade students receiving this curriculum did not answer, on a consistent basis, more problems correctly than those receiving a traditional curriculum. However, the lack of statistical significance may be a result of the small sample size (n = 14), as the results were consistent with the hypothesis (i.e., in the number and operations strand a higher percent of correct answers were found for sixth-grade students with learning disabilities educated with the standards-based curriculum [25.6% vs. 14.6%] and a lower percent of unanswered questions [29.9% vs. 40.6%]). The same was true for comparing across curricula in terms of the algebra strand and correct answers (30.8% vs. 20.7%, standards-based vs. traditional, respectively), although the frequency of answered questions was similar (31.8% vs. 30.3%).

Statistically significant relationships were found between the seventh grade for curricula with regard to answering questions correctly and not answering questions (i.e., leaving them blank). For the number and operations strand, a higher percent of correctly answered questions was found by students with learning disabilities who received the standards-based questions (54.6% vs. 32.9%, p < .01), as well as a lower percent of unanswered questions (8.3% vs. 21.9%, p < .001). The same was true for unanswered questions in the algebra strand (29.5% vs. 49.7%, p < .001). However, similar frequencies for correctly answered problems in the algebra strand were found between students receiving the two curricula (25.5% vs. 25.0%).

Chi Square Tests between curricula and error codes could not be completed because multiple cells had less than 5, and frequently less than 1 (i.e., low rates of errors coded as basic fact, wrong operation, place value, and for number and operations questions—no algebra). Hence, the presence or lack of statistically significant associations between curricula and error codes is unknown. However, a visual analysis of the frequencies showed few differences for sixth grade students for the number and operations strand and seventh grade for the algebra strand (refer to Tables 1 and 2). For the sixth grade algebra strand questions, notable frequency differences occurred in the error codes of defective algorithm, defective algorithm multi-step, and no conceptual understanding (refer to Table 2). More errors were coded as defective algorithm and defective algorithm multi-step by students who received the traditional curriculum, and more errors were coded as no algebra and no conceptual understanding for students who received the standards-based curriculum. Thus, although students who received the standards-based curriculum answered the algebra problems correct more often, when the answers were incorrect, students educated
with the traditional curriculum appeared to have a better sense of how to approach the problem. The opposite was found in the seventh grade number and operations problems. More errors were coded as no conceptual understanding for students who received the traditional curriculum (54.6% vs. 48%) and more were coded as defective algorithm for those educated with the standards-based curriculum (22.0% vs. 10.9%) (refer to Table 2).

Limitations

Many of the limitations of this research mirrored those of Jitendra and Kameenui (1996). For one, no response (i.e., answer left blank) was coded as 99–missing data, yet no response might indicate additional struggles not captured in this simple notion, such as the previously mentioned language difficulties or a lack of self-confidence (Babbitt, 1990; Quintero, 1984). No response to a problem could indicate a student did not know how to answer the problem, did not have time to answer the problem, or did not want to answer the problem, in addition to previously mentioned considerations. Furthermore, error analysis, as it was done here, only focused on incorrect responses, and ignored students' errors and incorrect strategies when they still managed to answer the question correctly (Jitendra & Kameenui). In addition, not all students provided work even though directed to in the instructions. Thus, error codes were derived from whatever was available, even if it was only the incorrect answer. This might have led to an incorrect interpretation of errors (Engelhardt, 1977). Finally, there were few students with learning disabilities, as the research was conducted in inclusive mathematics classes. Related, the research focused exclusively on students with language-based learning disabilities as students with learning disabilities were not educated in inclusive classes in any of the schools in the research project. Similarly, given the small n data could not be disaggregated for students with reading-based learning disabilities and learning disabilities in written expression; only two students had a learning disabilities in written expression in sixth-grade and four in seventh-grade.

Future Directions

Future research should continue to explore the errors students with disabilities make on open-ended problem-solving questions. Research should extend beyond whether students correctly or incorrectly answer questions, but also how they are answering open-ended problems. Understanding where students go wrong and the misconceptions they bring holds great potential for improving teacher planning and instruction. Additional research should include a larger population of students with learning disabilities as well as students with a mathematics-based learning disability. Unfortunately students with mathematics-based learning disabilities were not included in this study since students with such a disability were not educated in inclusive classrooms but rather in self-contained special education rooms in all four participating school districts. Furthermore, research asking students to do “think-alouds” when solving open-ended problems holds the potential to further capture the understandings, misconceptions and errors of students and lead to a greater understanding of student learning (Maccini & Hughes, 2000). With greater understanding of student struggles there is the possibility to develop better and more effective interventions, and hence ultimately improve student outcomes in school and beyond.

Implications

One main implication for this research is that students need greater help in understanding how to read, interpret, and solve open-ended problem-solving questions. The low rates of success coupled with the high frequency error codes that reflect a lack of understanding make it apparent that students with learning disabilities need either more instruction in problem solving or better instruction in problem solving. In other words, instruction need to be better in quality or quantity. Although a clear picture was not ascertained through the data, the findings suggest the beneficial nature of receiving the standards-based curriculum in terms of solving open-ended problems (i.e., answering more correctly and attempting more problems for the seventh-grade in the number and operations strand and more correct answers in the algebra strand). While these effects were not noted in the sixth grade data, perhaps it is just too short of time in such a curriculum, thus accounting for the differences in the seventh grade when students are in their second year with the standards-based curriculum.

References


Error Analysis and Curricula


Emily C. Bouck is an associate professor in the Department of Counseling, Educational Psychology, and Special Education at Michigan State University.

Mary K. Bouck is a retired public school superintendent, an adjunct instructor for the Teacher Education Program at Michigan State University and a consultant on mathematics education.

Gauri S. Joshi is a volunteer research assistant in the Special Education Program at Portland State University.

Linley Johnson was an undergraduate research trainee at the time of data collection; currently she is a high school mathematics teacher.

Please send correspondence to Emily C. Bouck, ecb@msu.edu.